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MA111 - Engineering Mathematics - II
Problem Sheet - 4

Power Series

1. In each of the following cases, determine the values of x for which the power series converges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{n^n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2 x^n}{(2n)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n n 2^n x^n$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{n3^n}$

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{n(\log n)^2}$

(f) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{10^n (x-10)^n}{n!}$

2. For each of the following power series determine the interval and radius of convergence.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(-3)^{2+n}(n^2+1)}(4x-12)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^{2n+1}}{4^{3n}}(2x+17)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{(2n+1)!}(x-2)^n$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{4^{1+2n}}{5^{n+1}}(x+3)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{n}(4x-1)^{n-1}$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{6^{1-n}}{(-2)^{3-2n}}(x+4)^n$

3. Write the given function as a power series and give the interval of convergence.

(a) $f(x) = \dfrac{x}{1-8x}$

(b) $f(x) = \dfrac{-12x^2}{1+6x^7}$

(c) $f(x) = \dfrac{x^7}{8+x^3}$

(d) $f(x) = \dfrac{\sqrt[5]{x^2}}{4-3x^2}$

4. Give a power series representation for (the derivative of) the following function.

(a) $g(x) = \dfrac{x^{10}}{2-x^2}$

(b) $g(x) = \dfrac{9x^5}{1+3x^6}$

5. Give a power series representation for (the integral of) the following function.

(a) $h(x) = \dfrac{7x}{3-6x}$

(b) $h(x) = \dfrac{x^4}{2+x^9}$

6. Make up a power series whose interval of convergence is

   (a) $(-3, 3)$ (b) $(-2, 0)$ (c) $(1, 5)$

7. (**Uniqueness of convergent power series**.) Show that if two power series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ are convergent and equal for all values of x in an open interval $(-c, c)$, then $a_n = b_n$ for all n.

8. Find the sum of the series $\sum_{n=0}^{\infty} n^2/2^n$.
   (*Hint: To find the sum of this series, express* $1/(1-x)$ *as a geometric series, differentiate both sides of the resulting equation with respect to x, multiply both sides of the result by x, differentiate again, multiply by x again, and set x equal to* $1/2$.)

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